Math 119 Chapter 17 **Name:** ______________________

Definition 1. *Bernoulli Trials are trials having these properties:*

1. *Only two outcomes are possible.*
2. *When Bernoulli Trials are repeated the trials are independent events.*
3. *When Bernoulli Trials are repeated the probability of a success on any one trial is the same for each trial.*

Geometric Probability Model

Used to find the probability that the 1st success is on a specific trial, x.

$$\begin{aligned} p &= \text{the probability of success} \\ q &= 1 - p = \text{the probability of failure} \\ X &= \text{the random variable representing the number of} \\ &\qquad \text{Bernoulli Trials until the first success is reached.} \\ x &= \text{a specific value of } X \end{aligned}$$

$$\boxed{P(X = x) \;=\; q^{x-1} \cdot p \qquad \textbf{Geometric Formula}}$$

1. A computer chip manufacturer rejects 3% of the chips produced because they fail presale testing. A quality control engineer examines randomly selected chips from the factory's latest output stacks. Assume the results are independent of one another.

   (a) What's the probability that the third chip she tests is the first bad one she finds?

   (b) Find the probability that the first bad chip found is found on the third or the fourth chip she examines.

   (c) Find the probability that the first bad chip found is found in one of the first three chips she tests.

Binomial Probability Model

Used to find the probability of x successes in n Bernoulli Trials.

$p =$ the probability of success

$q = 1 - p =$ the probability of failure)

$X =$ the random variable representing the number of successes found in n trials.

$$\boxed{P(X = x) \;=\; \left({}_{n}C_{x}\right) \cdot p^{x} \cdot q^{n-x} \qquad \textbf{Binomial Formula}}$$

$$\text{Where } \left({}_{n}C_{x}\right) = \frac{n!}{x!(n-x)!}$$

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2. A computer chip manufacturer rejects 3% of the chips produced because they fail presale testing. A quality control engineer examines 5 randomly selected chips from the factory's latest output stacks. Assume the results are independent of one another.

(a) Find the probability that there is one defective chip in the sample.

(b) Find the probability that there are two defective chips in the sample.

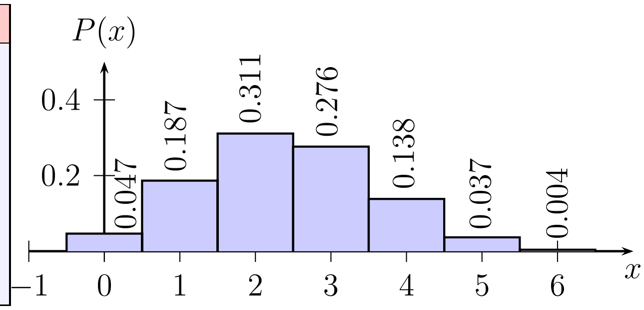
(c) Find the probability that there is at least one defective chip in the sample.

(d) Find the probability that there is between 2 and 4 defective chips in the sample.

Example: Assume that a procedure yields a binomial probability model with a trial repeated $n = 6$ times. Suppose the probability of success on a single trial is $p = 0.40$. Then, the probability model can be described with either the Binomial Formula, a table or a probability histogram.

$$P(X = x)$$
$$= \left({}_nC_x\right) \cdot p^x \cdot q^{n-x}$$
$$= \left({}_6C_x\right) \cdot (0.4)^x \cdot (0.6)^{6-x}$$

x	$P(X = x)$
0	0.047
1	0.187
2	0.311
3	0.276
4	0.138
5	0.037
6	0.004



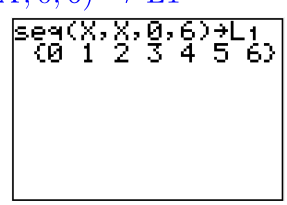
3. Use the model to find the probability of exactly $x = 3$ successes in $n = 6$ trials.

4. Find the probability of at least $x = 3$ successes in $n = 6$ trials.

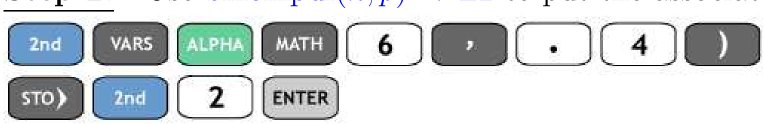
5. Find the probability of at most $x = 3$ successes in $n = 6$ trials.

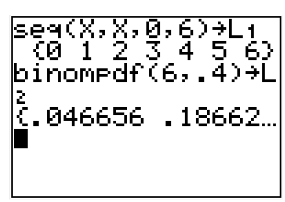
6. Calculator: Place the model in your calculator's list 1 (L1) and list 2 (L2).

Step 1: Put the numbers 0 through 6 in L1. Use seq$(X, X, 0, 6) \rightarrow$ L1



Step 2: Use binompdf$(n, p) \rightarrow$ L2 to put the associated probabilities in L2.

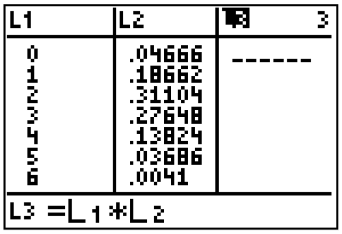


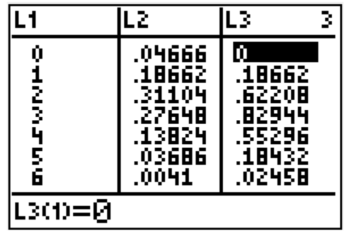


7. Find the expected value of the model, $\mu = E(X)$, using the formula

$$E(X) = \sum x \cdot P(X = x)$$

Remember, this means multiplying the entries in L1 by the entries in L2, then summing the products.





8. Find the expected value of the model, $\mu = E(X)$, using the formula $E(X) = n \cdot p$. What do you notice?

9. Find the standard deviation of the binomial model, σ, using the formula $\sigma = \sqrt{n \cdot p \cdot q}$

More Calculator Facts — Binomial Model

Your calculator has two binomial functions in its list of distributions (probability models). These are the $\boxed{\text{binompdf}}$ and $\boxed{\text{binomcdf}}$ functions.

- pdf stands for probability distribution function and gives the probability $P(X = x)$
- cdf stands for cumulative distribution function and gives the probability $P(X \leq x)$

Both functions take can take 3 input arguments: n, p, and x, each separated by a comma.

	TI–83/84
$P(X = x)$	binompdf(n, p, x)
$P(X \leq x)$	binomcdf(n, p, x)
$P(X < x)$	binomcdf$(n, p, x - 1)$
$P(X > x)$	$1 -$ binomcdf(n, p, x)
$P(X \geq x)$	$1 -$ binomcdf$(n, p, x - 1)$

Let $x = 3$	TI–83/84	
$P(X = 3)$	binompdf$(6, .4, 3)$	0.27648
$P(X \leq 3)$	binomcdf$(6, .4, 3)$	0.8208
$P(X < 3)$	binomcdf$(6, .4, 2)$	0.54432
$P(X > 3)$	$1 -$ binomcdf$(6, .4, 3)$	0.1792
$P(X \geq 3)$	$1 -$ binomcdf$(6, .4, 2)$	0.45568

10. A certain tennis player makes a successful first serve 70% of the time. Assume that each serve is independent of the others. If she serves 5 times, what's the probability she gets

(a) All 5 serves in?

(b) Exactly 4 serves in?

(c) At least 4 serves in?

(d) No more than 2 serves in?

11. Suppose the tennis player serves 100 times in a match (#29).

(a) What are the mean and standard deviation of the good first serves expected?

(b) Verify that you can use a Normal model to approximate the distribution of the number of good first serves.

(c) Use the 68–95–99.7 Rule to describe the distribution.

(d) Using the Normal Model, what's the probability she makes <u>at least 65</u> first serves?

(e) Using the Normal Model, what's the probability she makes <u>not more than 75</u> first serves?

(f) Using the Normal Model, what's the probability she makes <u>between 63 and 72</u> first serves?

12. A basketball player has made 70% of his foul shots during the season. Assuming the shots are independent, find the probability that in tonight's game he

(a) misses for the first time on his sixth attempt.

(b) makes his first basket on his fourth shot.

(c) makes his first basket on one of his first three shots.